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Minimization Implementation of Fuzzy Logic to Optimize a Cashew Nut Production using Simplex-Duality Theory

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ABSTRACT

Production control of cashew nuts greatly affects the profits earned by the company. When cashew nut raw materials exceed production needs, it results in storing cashews for an extended period, causing the cashew products to lose freshness, and the quality of processed cashews deteriorates. If the raw materials are damaged, the manufacturer must acquire additional costs to procure raw materials again. Another issue is that cashew nut raw materials are seasonal products and are not always available.
To solve these problems, a calculation method is needed to control optimal production inventory to minimize the company's expenditures. The method used is to formulate the problem into a Fuzzy Linear Programming mathematical model using a combination of methods: the Simplex algorithm and Duality Theory. The case implementation focuses on the uncertainty of cashew nut production quantities outside the harvest season. Moreover, the calculation of an optimal production quantity to minimize resulting losses is needed. The output generated is an optimal production prediction using the combination of the Simplex Algorithm and Duality Theory in solving Fuzzy Linear Programming.

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1. INTRODUCTION

The problem faced by the cashew processing industry is the seasonal harvest of raw cashew nut products, resulting in the accumulation of raw materials during one season, thus necessitating the optimization of proper raw material storage. This leads to fewer fresh products and lower-quality cashew products. Efforts made include processing cashews to optimize the stored raw materials in such conditions. Processed varieties include sweet and spicy, chocolate-flavored cashews, and more. The primary goal of a company is to achieve profitability, both economically, socially, and sustainably [1]. One of the factors affecting a company's profitability is optimal raw material management. Raw material inventory refers to the stock of items used in the production process [2]. Excessive raw material inventory not only leads to stockpiles but also increases storage costs. Conversely, if there is a shortage of inventory or damaged raw materials, it can result in additional procurement costs and disrupt the production process, potentially leading to a loss of consumer trust. In cases of uncertainty regarding consumer order demand, there is uncertainty in the quantity of goods to be produced. Hence, a calculation formula is needed using Fuzzy Linear Programming (FLP) modeling.

The research conducted by Lia Primadani and others focused on the production efficiency of UKM Souvenir businesses using Fuzzy Linear Programming with the Simplex fuzzy method. The solution using the Simplex fuzzy method involves several steps, including calculating the lower and upper optimal bounds through the maximization simplex method, modifying the initial equations by adding the variable λ and resolving it with the two-phase simplex method. The results of the simulation program show an estimated profit amounting to Rp 8,740,375.00 using the Linear Programming equation and Rp 9,510,003.00 using the Fuzzy Linear Programming equation, with a fuzzy membership function value of 0.5 [3]. Martini researched the production of hijabs using Fuzzy Linear Programming to determine the optimal quantity for each model and maximize profits. The Simplex method was employed to calculate the basic variables and the objective function. Using conventional Linear Programming (LP) would result in maximum profit by producing 18 pieces of fabric hijabs and 4 pashmina hijabs, generating a profit of Rp. 830,000 per day. On the other hand, using FLP would yield maximum profit by producing 21 pieces of fabric hijabs and 4 pashmina hijabs, resulting in a profit of Rp. 934,974 per day without violating any constraints. Therefore, it can be concluded that FLP can achieve higher profits compared to LP [4].

The search for an optimal solution in Fuzzy Linear Programming (FLP) with the objective function of minimization has been outlined using the Simplex method, The Big M method, or the Two-Phase Technique [5]. However, the complexity of calculations in each iteration makes it challenging to obtain an optimal solution. In [6], the concept of Duality Theory was introduced in solving Linear Programming models to perform sensitivity analysis and search for optimal solutions. Meanwhile, in the theory of Primal-Dual Linear Programming, the dual linear program is useful for reevaluating the simplex table in the primal linear program. Furthermore, several studies [7, 8, 9, 10] have applied combinations of these Primal-Dual methods in solving Fuzzy Linear Programming with various cases. Therefore, based on this background, the researcher aims to develop and implement a combination of the Simplex Algorithm and Duality Theory in solving the Fuzzy Linear Programming (FLP) model for raw material inventory control.

2. RESEARCH METHOD

The stages conducted of the research method are Observation and Literature Review, Data Collection, FLP Formulation, and the Search for the Optimal FLP Solution using a combination of methods. Here is an explanation of each step:

2.1. Observation and Study Literature

The observation was conducted in the Micro, Small, and Medium Enterprises (UMKM) industry that produces processed cashews in Wonogiri Regency. The literature study focused on Fuzzy Linear Programming theory and the Simplex-Duality Theory method.

2.2. Data Collection

The obtained data consists of raw materials used for five types of processed cashews (Roasted Cashews, Sweet-Spicy Cashews, Honey-Sesame Cashews, Chocolate Cashews, and Cashew Sambal), namely cashews, onions, chili peppers, sugar, salt, lime leaves, oil, honey, sesame seeds, DCC (Dark Compound Chocolate), cocoa powder, and galangal. The quantities of raw materials used in each production are listed in Table 1. Meanwhile, the prices for each raw material per kilogram for a single production are listed in Table 2. The prices of raw materials are uncertain because they follow market prices, especially the price of raw cashews, which is highly dependent on the harvest season and typically more expensive during festive seasons. In addition, the quantities of raw materials and production volumes are made on a scale according to the obtained data.

Table 1. Raw Material Requirements and Production Quantity in One Production Cycle

Varian	Raw Material (kg)												Production
	Cashews	Garlic	Chili	Sugar	Salt	Leaf Orange	Oil	Honey	Sesame	DCC	Chocolate Powder	Aromatic Gingers	
Baked Cashews	9.7-10												10-10.25
Sweet Spicy	9.7-10		0.9-1	1-1.1	0.25-0.4	0.1-0.2	0.8-1						10-10.25
Sesame Honey	9.7-10			0.3-0.4			0.8-1	0.15-0.2	3-3.5				10-10.25
Chocolate	9.7-10			0.5-0.7						2-2.2	1-1.5		10-10.25
Cashews Sambal	9.7-10	0.5-0.6	2.3-2.5	2.5-2.6	0.25-0.3	0.1-0.2						0.02-0.03	10-10.25

Table 2. Purchase Price per kilogram of Raw Materials for Cashew Processing

Types of Raw Materials	Price/kg
Cashews	Rp. 100.000,- until Rp. 120.000,-
Garlic	Rp. 32.000,- until Rp. 35.000,-
Chili	Rp. 42.000,- until Rp. 46.000,-
Sugar	Rp. 16.000,- until Rp. 20.000,-
Salt	Rp. 12.000,- until Rp. 15.000,-
Leaf Orange	Rp. 55.000,- until Rp. 65.000,-
Oil	Rp. 20.000,- until Rp. 30.000,-
Honey	Rp. 85.000,- until Rp. 100.000,-
Sasame	Rp. 50.000,- until Rp. 60.000,-
DCC (Dark Compound Chocolate)	Rp. 100.000,- until Rp. 120.000,-
Chocolate Powder	Rp. 46.000,- until Rp. 56.000,-
Aromatic Gingers	Rp. 17.000,- until Rp. 20.000,-

The data obtained in Table 1 and Table 2 indicate uncertain data. It makes Fuzzy Linear Programming (FLP) as a suitable model to solve the problem in minimizing the cost of raw materials for processed cashews considering the existing constraints.

2.3. Fuzzy Linear Programming Formulation

The next stage is formulating the optimization problem into Fuzzy Linear Programming with the objective function of minimization.

2.4. Searching for the optimal FLP Solution using a combination of methods

The last stage is searching for the optimal solution in Fuzzy Linear Programming using a combination of the Simplex-Duality Theory method. In addition, conducting an optimality test by comparing the solutions from several previous methods is also the activity in this stage.

3. RESULTS AND ANALYSIS

The main focus of this research is to find the optimal solution for the quantity of raw material inventory by implementing a combination of the Simplex Algorithm and Duality Theory in Fuzzy Linear Programming. The following are the stages involved:

a. Formulating the problem of processing cashew products with an uncertain production quantity into a Fuzzy Linear Programming model.
b. Implementing a combination of the Simplex Algorithm and Duality Theory in solving Fuzzy Linear Programming.

3.1. Fuzzification Process Using Fuzzy Membership Functions

To Solve the problem of determining the quantity of raw materials that should be procured to minimize purchasing costs while considering the uncertain production due to seasonal sales, Fuzzy Linear Programming (FLP) is required by representing numbers as fuzzy variables. All the data presented in Table 1 can be expressed in the form of fuzzy numbers with their membership functions.

a. The following is the membership function for a single production using cashew raw materials.
   1. If the fuzzy number to represent the cashew raw material required for the production of baked cashews is $\widetilde{B1}$, then the fuzzy number $\widetilde{B1}$ = (9.5, 9.7, 10, 10.2) where $a_1 = 9.5 - 9.7$ and $a_2 = 10 - 10.2$ Therefore, the membership function ($\mu_{\widetilde{B1}}$)is expressed as follows:

$$\mu_{\widetilde{B1}} = \begin{cases} \frac{x+0.2-9.7}{0.2}, & x \in [9.5,9.7] \\ 1, & \in [9.7,10] \\ \frac{-x+10+0.2}{0.2}, & \in [10,10.2] \\ 0, & etc \end{cases} \rightarrow \mu_{\widetilde{B1}} = \begin{cases} \frac{x-9.5}{0.2}, & x \in [9.5,9.7] \\ 1, & \in [9.7,10] \\ \frac{-x+10.2}{0.2}, & \in [10,10.2] \\ 0, & etc \end{cases} \quad (1)$$

The membership function of $\widetilde{B1}$ is depicted in the following Figure 1:

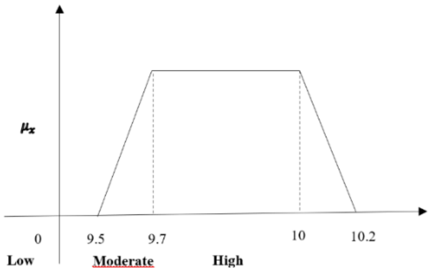

Figure 1. Membership Function of $\widetilde{B1}$ for Roasted Cashews

2. If the fuzzy number to represent the cashew raw material required for the production of sweet spicy cashews is $\widetilde{B2}$, then the fuzzy number $\widetilde{B2}$ = (9.5, 9.7, 10, 10.2) where $a_1 = 9.5 - 9.7$ and $a_2 = 10 - 10.2$ Therefore, the membership function $\mu_{\widetilde{B2}}$ is expressed as follows:

$$\mu_{\widetilde{B2}} = \begin{cases} \frac{x+0.2-9.7}{0.2}, & x \in [9.5,9.7] \\ 1, & \in [9.7,10] \\ \frac{-x+10+0.2}{0.2}, & \in [10,10.2] \\ 0, & etc \end{cases} \rightarrow \mu_{\widetilde{B2}} = \begin{cases} \frac{x-9.5}{0.2}, & x \in [9.5,9.7] \\ 1, & \in [9.7,10] \\ \frac{-x+10.2}{0.2}, & \in [10,10.2] \\ 0, & etc \end{cases} \quad (2)$$

The membership function of $\widetilde{B2}$ is depicted in the following Figure 2:

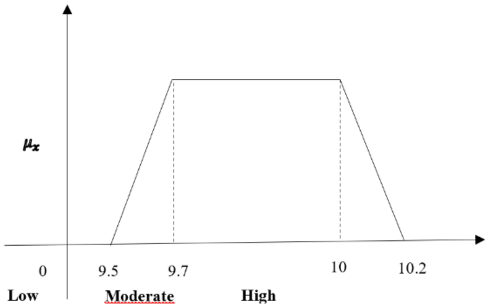

Figure 2. Membership Function of $\widetilde{B2}$ for Sweet Spicy Cashews

In the same manner, formulations are carried out to produce honey-sesame cashews, chocolate cashews, and cashew sambal.

b. The following is the membership function for a single production using garlic raw materials.
   1. If the fuzzy number to represent the garlic raw material required for the production of cashews sambal is $\widetilde{C5}$, then the fuzzy number $\widetilde{C5}$ = (0.4, 0.5, 0.6, 0.7) where $a_1 = 0.4 - 0.5$ and $a_2 = 0.6 - 0.7$ Therefore, the membership function $\mu_{\widetilde{C5}}$ is expressed as follows:

$$\mu_{\widetilde{C5}} = \begin{cases} \frac{x+0.5-0.1}{0.1}, & x \in [0.4,0.5] \\ 1, & \in [0.5,0.6] \\ \frac{-x+0.6+0.1}{0.1}, & \in [0.6,0.7] \\ 0, & etc \end{cases} \rightarrow \mu_{\widetilde{C5}} = \begin{cases} \frac{x+0.4}{0.1}, & x \in [0.4,0.5] \\ 1, & \in [0.5,0.6] \\ \frac{-x+0.7}{0.1}, & \in [0.6,0.7] \\ 0 & etc \end{cases} \quad (3)$$

The membership function of $\widetilde{C5}$ is depicted in the following Figure 3:

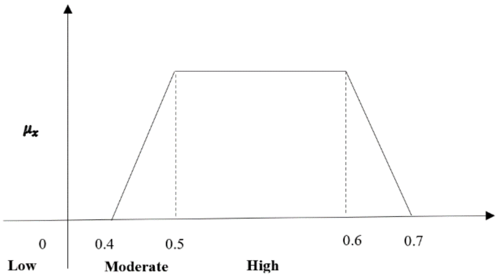

Figure 3. Membership Function of $\widetilde{C5}$ for Cashews Sambal

In the same manner, all the data can be expressed as symmetric trapezoidal fuzzy numbers, as shown in Table 3 below:

Table 3. Data in Symmetric Fuzzy Number Form

Varian	Raw Materials												Production
	Cashews	Garlic	Chili	Sugar	Salt	Leaf Orange	Oil	Honey	Sesame	DCC	Chocolate Powder	Aromatic Gingers	
Baked Cashews	(9.7,10, 0.15, 0.15)												(10,10.25, 0.125,0.125)
Sweet Spicy	(9.7,10, 0.15, 0.15)		(0.9,1, 0.05,0.05)	(1,1.1, 0.05, 0.05)	(0.3,0.4, 0.05,0.05)	(0.1,0.2,0.05,0.05)	(0.8,1,0.1,0.1)						(10,10.25, 0.125,0.125)
Sesame Honey	(9.7,10, 0.15, 0.15)			(0.3,0.4,0.05,0.05)			(0.8,1,0.1,0.1)	(0.15,0.2,0.025, 0.025)	(3,3.5,0.25,0.25)				(10,10.25, 0.125,0.125)
Chocolate	(9.7,10, 0.15, 0.15)			(0.5,0.7,0.1,0.1)						(2,2.2,0.1,0.1)	(1,1.5,0.25,0.25)		(10,10.25, 0.125,0.125)
Cashews Sambal	(9.7,10, 0.15, 0.15)	(0.5,0.6, 0.05, 0.05)	(2.3,2.5,0.1,0.1)	(2.5,2.6,0.05,0.05)	(0.25,0.3,0,025, 0.025)	(0.1,0.2,0.05,0.05)						(0.02,0.03, 0.005,0.005)	(10,10.25, 0.125,0.125)
Cost/kg	(100,120, 10,10)	(32,35,1.5,1.5)	(42,46,2,2)	(16,20,2,2)	(12,15,1.5,1.5)	(55,65,5,5)	(20,30,5,5)	(85,100, 7.5,7.5)	(50,60,5,5)	(100, 120,10,10)	(46,56,5,5)	(17,20.1.5, 1.5)	

3.2. Formulation of the Fuzzy Linear Programming Model

The problem in table 3 can be formulated into a mathematic model. The steps in creating a Fuzzy Linear model are as follows:

1. Determining Decision Variables.

   In this research, the selected decision variables are in the form of symmetric trapezoidal fuzzy numbers, which are:

   $\widetilde{x_1}$ = amount of cashew raw material required
   $\widetilde{x_2}$ = amount of $garlic$ raw material required
   $\widetilde{x_3}$ = amount of chili raw material required
   $\widetilde{x_4}$ = amount of sugar raw material required
   $\widetilde{x_5}$ = amount of salt raw material required
   $\widetilde{x_6}$ = amount of leaf orange raw material required
   $\widetilde{x_7}$ = amount of oil raw material required
   $\widetilde{x_8}$ = amount of honey raw material required
   $\widetilde{x_9}$ = amount of sesame raw material required
   $\widetilde{x_{10}}$ = amount of DCC raw material required
   $\widetilde{x_{11}}$ = amount of chocolate powder raw material required
   $\widetilde{x_{12}}$ = amount of aromatic gingers raw material required
   $\widetilde{Z}$ = objective function for minimizing costs

2. Determining Constrains.

   To obtain the appropriate amount of raw material for producing the five types of cashew products, the constraint faced is the optimal production quantity that must be achieved in a single production process. The constraints for each production quantity of cashew products to be achieved are as follows:

   a. Baked cashews production

   $$(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \geq (10,10.25, 0.125,0.125) \quad (4)$$

   b. Sweet spicy production

   $$(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.9,1, 0.05,0.05) \otimes \widetilde{x_3} \oplus (1,1.1, 0.05, 0.05) \otimes \widetilde{x_4} \oplus (0.3,0.4,0.05,0.05) \otimes \widetilde{x_5} \oplus (0.1,0.2,0.05,0.05) \otimes \widetilde{x_6} \oplus (0.8,1,0.1,0.1) \otimes \widetilde{x_7} \geq (10,10.25, 0.125,0.125) \quad (5)$$

   c. Sesame honey production

   $$(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.3,0.4,0.05,0.05) \otimes \widetilde{x_4} \oplus (0.8,1,0.1,0.1) \otimes \widetilde{x_7} \oplus (0.15,0.2,0.025,0.025) \otimes \widetilde{x_8} \oplus (3,3.5,0.25,0.25) \otimes \widetilde{x_9} \geq (10,10.25, 0.125,0.125) \quad (6)$$

   d. Chocolate production

   $$(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.5,0.7,0.1,0.1) \otimes \widetilde{x_4} \oplus (2,2.2,0.1,0.1) \otimes \widetilde{x_{10}} \oplus (1,1.5,0.25,0.25) \otimes \widetilde{x_{11}} \geq (10,10.25, 0.125,0.125) \quad (7)$$

e. Cashews Sambal production
$(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.5,0.6, 0.05, 0.05) \otimes \widetilde{x_2} \oplus (2.3,2.5,0.1,0.1) \otimes \widetilde{x_3} \oplus (2.5,2.6,0.05,0.05) \otimes \widetilde{x_4} \oplus (0.25,0.3,0,025,0.025) \otimes \widetilde{x_5} \oplus (0.1,0.2,0.05,0.05) \otimes \widetilde{x_6} \oplus (0.02,0.03,0.005,0.005) \otimes \widetilde{x_{12}} \geq (10,10.25, 0.125,0.125)$ (8)

3. Determining Goal.
The objective to be achieved is to minimize the cost of procuring raw materials for producing cashew products. Therefore, the objective function is formed from the purchasing cost of raw materials for each type of cashew product. Thus, the objective function for minimizing costs is as follows:
$$\tilde{Z} = (100,120, 10,10) \otimes \widetilde{x_1} \oplus (32,35,1.5,1.5) \otimes \widetilde{x_2} \oplus (42,46,2,2) \otimes \widetilde{x_3} \oplus (16,20,2,2) \otimes \widetilde{x_4} \oplus (12,15,1.5,1.5) \otimes \widetilde{x_5} \oplus (55,65,5,5) \otimes \widetilde{x_6} \oplus (20,30,5,5) \otimes \widetilde{x_7} \oplus (85,100,7.5,7.5) \otimes \widetilde{x_8} \oplus (50,60,5,5) \otimes \widetilde{x_9} \oplus (100,120,10,10) \otimes \widetilde{x_{10}} \oplus (46,56,5,5) \otimes \widetilde{x_{11}} \oplus (17,20.1.5,1.5) \otimes \widetilde{x_{12}} \quad (9)$$

4. Implementation Fuzzy Linear Programming (FLP) with the Mehar Method for Minimizing Processed Cashew Raw Material Costs.
The problem in the "Processed Cashew Production" industry in Wonogiri is a Fuzzy Linear Programming problem with coefficients of decision variables represented as symmetric trapezoidal fuzzy numbers. This problem will be solved using the Mehar method. The steps for its resolution are as follows:
   1. Using the first step of the Mehar method, the problem formulation in the "Processed Cashew Production" industry is expressed in the following equation:
   Minimize:
   $$\tilde{Z} = (100,120, 10,10) \otimes \widetilde{x_1} \oplus (32,35,1.5,1.5) \otimes \widetilde{x_2} \oplus (42,46,2,2) \otimes \widetilde{x_3} \oplus (16,20,2,2) \otimes \widetilde{x_4} \oplus (12,15,1.5,1.5) \otimes \widetilde{x_5} \oplus (55,65,5,5) \otimes \widetilde{x_6} \oplus (20,30,5,5) \otimes \widetilde{x_7} \oplus (85,100,7.5,7.5) \otimes \widetilde{x_8} \oplus (50,60,5,5) \otimes \widetilde{x_9} \oplus (100,120,10,10) \otimes \widetilde{x_{10}} \oplus (46,56,5,5) \otimes \widetilde{x_{11}} \oplus (17,20.1.5,1.5) \otimes \widetilde{x_{12}} \quad (10)$$

   With Constrains:

   $(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \geq (10,10.25, 0.125,0.125)$

   $(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.9,1, 0.05,0.05) \otimes \widetilde{x_3} \oplus (1,1.1, 0.05, 0.05) \otimes \widetilde{x_4} \oplus (0.3,0.4,0.05,0.05) \otimes \widetilde{x_5} \oplus (0.1,0.2,0.05,0.05) \otimes \widetilde{x_6} \oplus (0.8,1,0.1,0.1) \otimes \widetilde{x_7} \geq (10,10.25, 0.125,0.125)$

   $(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.3,0.4,0.05,0.05) \otimes \widetilde{x_4} \oplus (0.8,1,0.1,0.1) \otimes \widetilde{x_7} \oplus (0.15,0.2,0.025,0.025) \otimes \widetilde{x_8} \oplus (3,3.5,0.25,0.25) \otimes \widetilde{x_9} \geq (10,10.25, 0.125,0.125$

   $(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.5,0.7,0.1,0.1) \otimes \widetilde{x_4} \oplus (2,2.2,0.1,0.1) \otimes \widetilde{x_{10}} \oplus (1,1.5,0.25,0.25) \otimes \widetilde{x_{11}} \geq (10,10.25, 0.125,0.125)$

   $(9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.5,0.6, 0.05, 0.05) \otimes \widetilde{x_2} \oplus (2.3,2.5,0.1,0.1) \otimes \widetilde{x_3} \oplus (2.5,2.6,0.05,0.05) \otimes \widetilde{x_4} \oplus (0.25,0.3,0,025,0.025) \otimes \widetilde{x_5} \oplus (0.1,0.2,0.05,0.05) \otimes \widetilde{x_6} \oplus (0.02,0.03,0.005,0.005) \otimes \widetilde{x_{12}} \geq (10,10.25, 0.125,0.125$

   $\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4}, \widetilde{x_5}, \widetilde{x_6}, \widetilde{x_7}, \widetilde{x_8}, \widetilde{x_9}, \widetilde{x_{10}}, \widetilde{x_{11}}, \widetilde{x_{12}} \geq \tilde{0}$ (11)

   2. Using the second step of the Mehar method, problem (11) can be transformed into an equation as follows:
   Minimize:
   $$\Re(\widetilde{Z}) = \Re((100,120, 10,10) \otimes \widetilde{x_1} \oplus (32,35,1.5,1.5) \otimes \widetilde{x_2} \oplus (42,46,2,2) \otimes \widetilde{x_3} \oplus (16,20,2,2) \otimes \widetilde{x_4} \oplus (12,15,1.5,1.5) \otimes \widetilde{x_5} \oplus (55,65,5,5) \otimes \widetilde{x_6} \oplus (20,30,5,5) \otimes \widetilde{x_7} \oplus (85,100,7.5,7.5) \otimes \widetilde{x_8} \oplus (50,60,5,5) \otimes \widetilde{x_9} \oplus (100,120,10,10) \otimes \widetilde{x_{10}} \oplus (46,56,5,5) \otimes \widetilde{x_{11}} \oplus (17,20.1.5,1.5) \otimes \widetilde{x_{12}}) \quad (12)$$

   With Constrains:

   $\Re ((9.7,10, 0.15, 0.15) \otimes \widetilde{x_1}) \geq \Re (10,10.25, 0.125,0.125)$

   $\Re ((9.7,10, 0.15, 0.15) \otimes \widetilde{x_1} \oplus (0.9,1, 0.05,0.05) \otimes \widetilde{x_3} \oplus (1,1.1, 0.05, 0.05) \otimes \widetilde{x_4} \oplus (0.3,0.4,0.05,0.05) \otimes \widetilde{x_5} \oplus (0.1,0.2,0.05,0.05) \otimes \widetilde{x_6} \oplus (0.8,1,0.1,0.1) \otimes \widetilde{x_7}) \geq \Re (10,10.25, 0.125,0.125)$

$\Re$ ((9.7,10, 0.15, 0.15) $\otimes \widetilde{x_1} \oplus$ (0.3,0.4,0.05,0.05)$\otimes \widetilde{x_4} \oplus$ (0.8,1,0.1,0.1)$\otimes \widetilde{x_7} \oplus$ (0.15,0.2,0.025,0.025)$\otimes \widetilde{x_8} \oplus$ (3,3.5,0.25,0.25) $\otimes \widetilde{x_9}$) $\geq \Re$ (10,10.25, 0.125,0.125)

$\Re$ ((9.7,10, 0.15, 0.15) $\otimes \widetilde{x_1} \oplus$ (0.5,0.7,0.1,0.1)$\otimes \widetilde{x_4} \oplus$ (2,2.2,0.1,0.1) $\otimes \widetilde{x_{10}} \oplus$ (1,1.5,0.25,0.25)$\otimes \widetilde{x_{11}}$) $\geq \Re$ (10,10.25, 0.125,0.125)

$\Re$ ((9.7,10, 0.15, 0.15) $\otimes \widetilde{x_1} \oplus$ (0.5,0.6, 0.05, 0.05) $\otimes \widetilde{x_2} \oplus$ (2.3,2.5,0.1,0.1)$\otimes \widetilde{x_3} \oplus$ (2.5,2.6,0.05,0.05) $\otimes \widetilde{x_4} \oplus$ (0.25,0.3,0,025,0.025) $\otimes \widetilde{x_5} \oplus$ (0.1,0.2,0.05,0.05)$\otimes \widetilde{x_6} \oplus$ (0.02,0.03,0.005,0.005)$\otimes \widetilde{x_{12}}$) $\geq \Re$ (10,10.25, 0.125,0.125)

$$\Re(\widetilde{x_1}, \widetilde{x_2}, \widetilde{x_3}, \widetilde{x_4}, \widetilde{x_5}, \widetilde{x_6}, \widetilde{x_7}, \widetilde{x_8}, \widetilde{x_9}, \widetilde{x_{10}}, \widetilde{x_{11}}, \widetilde{x_{12}}) \geq \Re(\tilde{0}) \quad (13)$$

3. Using step 3 of the Mehar method, following the rules as follows:

$$\Re\left(\sum_{j=1}^{n} \tilde{c} \otimes \tilde{x_j} = \sum_{j=1}^{n} \Re(\tilde{c} \otimes \tilde{x_j})\right)\Re(\tilde{x_j})$$

and

$$\Re\left(\sum_{j=1}^{n} \widetilde{a_{ij}} \otimes \tilde{x_j}\right) = \sum_{j=1}^{n} \Re(\widetilde{a_{ij}} \otimes \tilde{x_j}) = \sum_{j=1}^{n} \Re(\widetilde{a_{ij}})\Re(\tilde{x_j}) \quad (14)$$

The equation (14) can be transformed into equation (16) as follows:
Minimize:

$\Re(\widetilde{Z})$=$\Re$(100,120, 10,10) $\Re(\widetilde{x_1})\oplus\Re$(32,35,1.5,1.5) $\Re(\widetilde{x_2})\oplus\Re$(42,46,2,2)$\Re(\widetilde{x_3})\oplus$ $\Re$(16,20,2,2)$\Re(\widetilde{x_4})\oplus\Re$(12,15,1.5,1.5)$\Re(\widetilde{x_5})\oplus\Re$(55,65,5,5)$\Re(\widetilde{x_6}) \oplus$ $\Re$(20,30,5,5)$\Re(\widetilde{x_7})\oplus\Re$(85,100,7.5,7.5)$\Re(\widetilde{x_8})\oplus\Re$(50,60,5,5)$\Re(\widetilde{x_9})\oplus\Re$ (100,120,10,10)$\Re(\widetilde{x_{10}})\oplus$ $\Re$(46,56,5,5) $\Re\widetilde{x_{11}} \oplus$ $\Re$(17,20.1.5,1.5) $\Re(\widetilde{x_{12}})$ (15)

With constrains:

$\Re$ (9.7,10, 0.15, 0.15)$\Re(\widetilde{x_1}) \geq \Re$ (10,10.25, 0.125,0.125)

$\Re$ (9.7,10, 0.15, 0.15) $\Re(\widetilde{x_1})\oplus\Re$(0.9,1, 0.05,0.05)$\Re(\widetilde{x_3})\oplus\Re$(1,1.1, 0.05, 0.05)$\Re(\widetilde{x_4})\oplus\Re$(0.3,0.4,0.05,0.05)$\Re(\widetilde{x_5})\oplus\Re$(0.1,0.2,0.05,0.05)$\Re(\widetilde{x_6})\oplus\Re$(0.8,1,0.1,0.1)$\Re(\widetilde{x_7}) \geq \Re$ (10,10.25, 0.125,0.125)

$\Re$(9.7,10, 0.15, 0.15)$\Re(\widetilde{x_1})\oplus\Re$(0.3,0.4,0.05,0.05)$\Re(\widetilde{x_4})\oplus\Re$(0.8,1,0.1,0.1)$\Re(\widetilde{x_7})\oplus$ $\Re$(0.15,0.2,0.025,0.025)$\Re(\widetilde{x_8})\oplus\Re$(3,3.5,0.25,0.25)$\Re(\widetilde{x_9}) \geq \Re$ (10,10.25, 0.125,0.125)

$\Re$(9.7,10, 0.15, 0.15)$\Re(\widetilde{x_1})\oplus\Re$(0.5,0.7,0.1,0.1)$\Re(\widetilde{x_4})\oplus\Re$(2,2.2,0.1,0.1)$\Re(\widetilde{x_{10}})\oplus$ $\Re$(1,1.5,0.25,0.25)$\Re(\widetilde{x_{11}}) \geq\Re$ (10,10.25, 0.125,0.125)

$\Re$(9.7,10, 0.15, 0.15)$\Re(\widetilde{x_1})\oplus$ $\Re$(0.5,0.6, 0.05, 0.05)$\Re(\widetilde{x_2})\oplus\Re$(2.3,2.5,0.1,0.1)$\Re(\widetilde{x_3})\oplus$ $\Re$(2.5,2.6,0.05,0.05)$\Re(\widetilde{x_4})\oplus$ (0.25,0.3,0,025,0.025) $\otimes \widetilde{x_5} \oplus$ (0.1,0.2,0.05,0.05)$\otimes \widetilde{x_6} \oplus$ $\Re$(0.02,0.03,0.005,0.005)$\Re(\widetilde{x_{12}}) \geq \Re$ (10,10.25, 0.125,0.125)

$$\Re(\widetilde{x_1}) \geq \Re(\tilde{0}),\ \Re(\widetilde{x_2}) \geq \Re(\tilde{0}), \Re(\widetilde{x_3}) \geq \Re(\tilde{0}), \Re(\widetilde{x_4}) \geq \Re(\tilde{0}), \Re(\widetilde{x_5}) \geq \Re(\tilde{0}), \Re(\widetilde{x_6}) \geq \Re(\tilde{0}), \Re(\widetilde{x_7}) \geq \Re(\tilde{0}), \Re(\widetilde{x_8}) \geq \Re(\tilde{0}), \Re(\widetilde{x_9}) \geq \Re(\tilde{0}), \Re(\widetilde{x_{10}}) \geq \Re(\tilde{0}), \Re(\widetilde{x_{11}}) \geq \Re(\tilde{0}), \Re(\widetilde{x_{12}}) \geq \Re(\tilde{0}) \quad (16)$$

4. Using step 4 of the Mehar method, because $\Re(\widetilde{A})$ is a real number, it is assumed $\Re(\widetilde{Z}) = Z, \Re(\widetilde{c_j}) = c_j$, $\Re(\widetilde{x_j}) = x_j$, $\Re(\widetilde{a_{ij}}) = a_{ij}$ $\Re(\widetilde{b}) = b_i$ and put $\Re(\widetilde{0}) = 0$ The problem in equation (15) can be transformed into equation (17):
Minimize:

$$Z = 110x_1 + 33.5x_2 + 44x_3 + 18x_4 + 13.5x_5 + 60x_6 + 25x_7 + 92.5x_8 + 55x_9 + 110x_{10} + 51x_{11} + 18.5x_{12}$$

With constrains:

$$9.85x_1 \geq 10.125$$
$$9.85x_1 + 0.95x_3 + 1.05x_4 + 0.35x_5 + 0.15x_{6\_} + 0.9x_7 \geq 10.125$$
$$9.85x_1 + 0.35x_4 + 0.9x_7 + 0.175x_8 + 3.25x_9 \geq 10.125$$
$$9.85x_1 + 0.6x_4 + 2.1x_{10} + 1.25x_{11} \geq 10.125$$
$$9.85x_1 + 0.55x_2 + 2.4x_3 + 2.55x_4 + 0.275x_5 + 0.15x_6 + 0.025x_{12} \geq 10.125$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0, x_6 \geq 0,$$
$$x_7 \geq 0, x_8 \geq 0, x_9 \geq 0, x_{10} \geq 0, x_{11} \geq 0, x_{12} \geq 0 \quad (17)$$

Equation (17) is a Linear Programming (LP) model for the minimization problem

5. Minimizing cases with more than 10 variables would be time-consuming and prone to significant calculation errors due to the complexity of the algorithm involved. Therefore, the researcher will apply a combination of the Simplex-Duality Theory methods to obtain the optimal solution. This method will transform the minimum Linear Programming (LP) case of equation (17) into a maximum LP case. As a result, the calculation process will be more straightforward because maximum cases use simpler algorithms. The steps are the initial Linear Programming (LP) in equation (17) obtained is referred to as the Primal LP Model. Using the Primal-Dual method, the Dual LP Model is derived as follows:

   Maximize:
   $Y = 10.125y_1 + 10.125y_2 + 10.125y_3 + 10.125y_4 + 10.125y_5$

   with constrains:

$$9.85y_1 + 9.85y_2 + 9.85y_3 + 9.85y_4 + 9.85y_5 \leq 110$$
$$0.55y_5 \leq 33.5$$
$$0.95y_2 + 2.4y_5 \leq 44$$
$$1.05y_2 + 0.35y_3 + 0.6y_4 + 2.55y_5 \leq 18$$
$$0.35y_2 + 0.275y_5 \leq 13.5$$
$$0.15y_2 + 0.15y_5 \leq 60$$
$$0.9y_2 + 0.9y_3 \leq 25$$

$$0.175y_3 \leq 92.5$$
$$3.25y_3 \leq 55$$
$$2.1y_4 \leq 110$$
$$1.25y_4 \leq 51$$
$$0.025y_5 \leq 18.5$$
$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0, y_5 \geq 0 \quad (18)$$

   where: y_i, i = 1,2,3,4,5, are variables formed by the results of the primal dual process.

3.2. Optimal Solution

The optimal solution for equation (18) is obtained using the simplex method (simplex table) as follows as shown in figure 4:

	X1	X2	X3	X4	X5		RHS	Dual
Maximize	10.125	10.125	10.125	10.125	10.125			
Constraint 1	9.85	9.85	9.85	9.85	9.85	<=	110	1.0279
Constraint 2	0	0	0	0	.55	<=	33.5	0
Constraint 3	0	.95	0	0	2.4	<=	44	0
Constraint 4	0	1.05	.35	.6	2.55	<=	18	0
Constraint 5	0	.35	0	0	.275	<=	13.5	0
Constraint 6	0	.15	0	0	.15	<=	60	0
Constraint 7	0	.9	.9	0	0	<=	25	0
Constraint 8	0	0	.175	0	0	<=	92.5	0
Constraint 9	0	0	3.25	0	0	<=	55	0
Constraint 10	0	0	0	2.1	0	<=	110	0
Constraint 11	0	0	0	1.25	0	<=	51	0
Constraint 12	0	0	0	0	.025	<=	18.5	0
Solution->	11.1675	0	0	0	0		113.0711	

Figure 4. Optimal Solution of Dual Linear Programming Problem (Maximization) Using POM Application

Based on the optimal table in Figure 4, the optimal solution is found in the dual column by reversing the value of constraint 1, resulting in the value of x1 as Z= 113.0711, x_1=1.0279, x_2=0, x_3=0, x_4=0, x_5=0, x_6=0, x_7=0, x_7=0, x_8=0, x_9=0, x_{10}=0, x_{11}=0, x_{12}=0.

The optimal solution, when solved using the Primal equation (16) with the Minimum Linear Programming (LP) approach, will be obtained as follows:

Variable	Status	Value
X1	Basic	1.0279
X2	NONBasic	0
X3	NONBasic	0
X4	NONBasic	0
X5	NONBasic	0
X6	NONBasic	0
X7	NONBasic	0
X8	NONBasic	0
X9	NONBasic	0
X10	NONBasic	0
X11	NONBasic	0
X12	NONBasic	0
surplus 1	Basic	0
surplus 2	Basic	0
surplus 3	NONBasic	0
surplus 4	Basic	0
surplus 5	Basic	0
Optimal Value (Z)		113.0711

Figure 5. Optimal Solution of Primal Linear Programming (LP) Model Using POM Application

4. CONCLUSION

The results of the primal linear programming (PL) model calculations in the "value" column indicate the optimal solution obtained: Z = 113.0711, x1 = 1.0279, x2 = 0, x3 = 0, x4 = 0, x5 = 0, x6 = 0, x7 = 0, x7 = 0, x8 = 0, x9 = 0, x10 = 0, x11 = 0, x12 = 0. Meanwhile, the results of the dual linear programming (PL Dual) model yield the optimal solution. The optimal values are in the dual column, with the reversed constraint being x1, and likewise, Z = 113.0711, x1 = 1.0279, x2 = 0, x3 = 0, x4 = 0, x5 = 0, x6 = 0, x7 = 0, x7 = 0, x8 = 0, x9 = 0, x10 = 0, x11 = 0, x12 = 0. From the obtained solution, it can be interpreted that by producing 11.1675 kg of roasted cashews, the total cost of raw materials spent will be minimized to Rp. 113.0711. The results in this case are based on real data, and the optimal values will follow the data (coefficients) entered into the Fuzzy Linear Programming model.

Therefore, from both the primal and dual models, the same answer is obtained. The conclusion that can be drawn is that the Simplex-Duality Theory method can be applied to the FLP Minimization case, resulting in an optimal solution. Furthermore an application based on Fuzzy Logic should be created to solve the case and find the result easier.

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