

USING MATLAB TO HELP STUDENTS UNDERSTAND GAUSS-JORDAN METHODS

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Abstract

There are many variation of Gauss-Jordan Calculator. Students can use it to solve various types of linear equations. But all of them just straightly show the result. There is no such calculator which shows the step by step to get the result. Therefore, the students cannot understand how to do Gauss-Jordan Elimination. One program that can be developed to make Gauss-Jordan calculator is matlab. This article examines matlab program which can help students to understand Gauss-Jordan Elimination methods. It tends to focus on the process to get solution. This program reveals with the step by step of operation in Gauss-Jordan to make reduced row-echelon form. Numbers of operations drawn in the solutions of linear simultaneous equations have also been showed. This paper has a propensity to appraise the performance of matlab program in solving linear equation problems with n variables while showing every steps of Gauss-Jordan Elimination. Further it has the potential to develop this program to be a calculator that can be access without opening matlab.

Key Word: Matlab, Gauss-Jordan, Elimination, steps operation

Abstrak

Calculator Gauss-Jordan adalah semacam program computer yang dapat menyelesaikan suatu permasalahan sistem persamaan linear dengan menggunakan Metode Eliminasi Gauss Jordan. Banyak sekali variasi calculator seperti itu di dunia maya. Saat ini siswa dapat dengan mudah menggunakan calculator tersebut untuk menyelesaikan sistem persamaan linear. Tetapi semua variasi calculator ini hanya langsung menunjukkan hasil akhirnya. Tidak ada calculator Gauss-Jordan yang menunjukkan langkah-langkah mendapatkan solusinya atau biasa dikenal dengan operasi baris elementer. Hal ini mengakibatkan siswa tidak dapat memahami bagaimana cara kerja metode Gauss-Jordan. Salah satu program computer yang dapat digunakan untuk membuat Gauss-Jordan calculator adalah matlab. Penelitian ini akan menggunakan program computer matlab untuk membuat calculator Gauss-Jordan yang selain menyelesaikan sistem persamaan linear, juga dapat menunjukkan operasi baris elementer yang dilakukan sehingga mendapatkan baris eselon tereduksi. Calculator Gauss-Jordan pada penelitian ini masih harus dibuka dalam program matlab, sehingga kedepannya diharapkan program ini dapat digunakan tanpa harus membuka matlab.

Kata kunci : Matlab, Gauss-Jordan, Eliminasi, Langkah Operasi

1. Introduction

In science and engineering, system of linear equations arises in various theoretical research and applications. Gauss-Jordan Elimination is well known technique to determine a common solution in linear algebra. This method asked the Linear Equation System to be Matrices. Then is will solve by making the matrix into reduced row-echelon form. The way to change the matrix into the reduced row-echelon form is not easy. We have to try several operation possibilities. Although solving linear equation system using Gauss-Jordan Methods is not easy, but this method allowed us to solve equation system for n variables. Therefore, Gauss-Jordan method is very important to be learned.

In the school, when solving systems of equations by using matrices, many teachers present a Gauss-Jordan elimination approach to row reducing matrices that can involve painfully tedious operations with fractions (Smith and Powell, 2011) as well as in the

Campus. In Sekolah Tinggi Teknologi Adisujipto (STTA) majoring Aeronautica Engineering, Linear Equation System is taught in Engineering Mathematic II of the second semester of 2016-2017 which related to Matrix. Based on the students' result in Final Test (2016-2017), many students did operations that do not lead to reduces row-echelon form and error while counting the operation to solve the Linear Equation system by Gauss-Jordan Method. They need tools to know what kind of operations need to get reduced row-echelon form and why it is so.

Gauss-Jordan calculator is one of the solutions to solve system of linear equation. Various type of this calculator will easily found in internet. But all of them just show the final result of common solution of linear equation systems. This calculator could be useful for application but not for learning. Students tend to use this calculator to solve linear equation systems. But when they asked to show how the steps to get the solution, they get confused. Therefore, for students in learning process Gauss-Jordan calculator is not really helpful.

This kind of calculator is developed by various computer languages programs. One of them is Matlab. Matlab (Matrix Laboratory) is a tool for numerical computation and visualization. The basic data element is a matrix, so this program could manipulates array-based data fast and correct. For engineering students, matlab is also familiar.

This study will make a program using Matlab to solve linear equation system by Gauss-Jordan Elimination. The program could show the steps to find the reduced row-echelon form in order to help students understand how to find the suitable operation. Next, the program will be showed to the students to get their opinion.

This program is very helpful for students to know what kind of operations should do to make the reduced row-echelon form. The students were very interested to this program. They thought it was awesome since this program is very dynamic. It can be used for any type of square matrix with any element of it. The most important things from this program is showing the steps of operation to get reduced row-echelon form.

2. Background Theory

2.1 System of Linear Equation

Systems of linear equations play an important and motivating role in the subject of linear algebra. In fact, many problems in linear algebra reduce to finding the solution of a system of linear equations.

Linear Equation is unknowns x_1, x_2, \dots, x_n is an equation that can be put in the standard form

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b \quad (1.1)$$

where a_1, a_2, \dots, a_n , and b are constants. The constant a_k is called the coefficient of x_k , and b is called the constant term of the equation.

A solution of the linear equation (1.1) is a list of values for the unknowns or, equivalently, a vector u in K^n , say

$$x_1 = k_1, \quad x_2 = k_2, \quad \dots, \quad x_n = k_n \quad \text{or} \quad u = (k_1, k_2, \dots, k_n)$$

such that the following statement (obtained by substituting k_i for x_i in the equation) is true:

$$a_1k_1 + a_2k_2 + \dots + a_nk_n = b$$

in such a case we say that u satisfies the equation.

A system of linear equations is a list of linear equations with the same unknowns. In particular, a system of m linear equations L_1, L_2, \dots, L_m in n unknowns x_1, x_2, \dots, x_n can be put in the standard form.

- b) All entries in the column above the pivotal 1 are equal to 0. Of the above, 3(b), while part of the definition of reduced row-echelon form, is not necessary for the system to be triangular.

Converting a system into reduced row-echelon form, we must do elementary row operations.

- 1) Multiply or divide a row by a nonzero scalar.
- 2) Add or subtract a multiple of a row to another row.
- 3) Swap two rows.

The end goal of Gauss Jordan Elimination is to convert the system to reduced row-echelon form. The permissible moves are the above three types of reversible operations. The computer program can use to figure out a general procedure that, starting with any system, can identify the appropriate sequence of permissible moves to reach our end goal. The choice of moves will obviously depend on the system we start with. If we want to make the process as algorithmically straightforward as possible, we ultimately do not have to think much about it. The operations that performed are done on the augmented matrix, but they are controlled by the coefficient matrix, in the sense that our decision of what operation to do at each stage is completely governed by the coefficient matrix. It does not ever actually need to look at the right-most column in the decision process for the operation.

2.3 Matlab

The computer program that can use to figure out algorithmic of Gauss Jordan Elimination is Matlab. MATLAB or Matrix Laboratory is a tool for numerical computation and visualization. The basic data element of this program is a matrix. It can manipulate array-based data and it is generally fast to write and run in Matlab program. Since Gauss-Jordan elimination used matrix to solve system of linear equation, so Matlab is suitable to be used. Matlab has several advantages over other methods or language which is its basic data element is the matrix. A simple integer is considered a matrix of one row and one column. Several mathematical operations that work on arrays or matrices are built-in to the Matlab environment. For example, cross-products, dot-products, determinants, inverse matrices.

Actually, Matlab's functionality can be greatly expanded by the addition of toolboxes. Ex: Excel link allows data to be written in a format recognized by Excel, Statistics Toolbox allows more specialized statistical manipulation of data (Anova, Basic Fits, etc). But in this study, the program has not expanded to other toolboxes. Therefore, it is just can be used by opening matlab.

Based on students interviewed after final Engineering Mathematics II test about system of linear equation, many of them said that the way to choose operation which suitable to change augmented matrix into reduced row-echelon form is the most difficult parts. The rest said that the counting operation that involved fraction made it difficult.

Therefore, this study promotes a dynamic syntax of matlab which can show the process of Gauss – Jordan Elimination for the students. In this program, students can input any number element of matrix to the program. They also can choose the form of square matrix. All the consideration led us to the following research question: “How Matlab program could help students to understand Linear Equation System using Gauss-Jordan Method?”

3. Research Method

Computer program Matlab to solve the system of linear equations involving its elementary row operations with the help of Gauss-Jordan elimination has been developed and is given below-

Table 1
The Algorithm of Matlab Program to solve Gauss-Jordan Elimination

```
clear all;
clc;
disp('METODE GAUSS');
n = input('Masukkan besar matrik : ');
fprintf('Matrik A dengan ukuran %d x %d',n,n);
disp(' ');
for i=1:n
for j=1:n
fprintf('A(%d,%d)', i,j);
A(i,j) = input(' ');
end
end

fprintf('Matrik B dengan ukuran %d x 1\n',n);

for i=1:n
fprintf('B(%d,1)',i);
B(i,1) = input(' ');
end
fprintf('nilai determinan A = %0.2f\n', det(A));
detA = det(A);
if detA ~= 0
M = [A B] ;
fprintf('PROSES METODE GAUSS\n');
[m,n]=size(M);
En = M;
disp (M)
end
for i=1:n-1
if M(i,i)==0
a=i;
while M(a,i)==0
a=a+1;
end
T=M(i,:);
```

```

M(i,:)=M(a,:);
M(a,:)=T
disp(['B' num2str(i) '<--> B' num2str(a)])
disp (M)
end
for j=i+1:n
    if M(j,i)~=0
        disp(['B' num2str(j) ' - (' num2str(M(j,i)) '/' num2str(M(i,i)) ')B' num2str(i)])
        M(j,:)=M(j,:)-((M(j,i)/M(i,i))*M(i,:));
        disp M
    end
end
end
if M(n,n)==0&M(n,n+1)~=0
    disp ('SPL tidak memiliki penyelesaian tunggal')
else
    for i=n:-1:2
        for j=i-1:-1:1
            disp(['B' num2str(j) ' - (' num2str(M(j,i)) '/' num2str(M(i,i)) ')B' num2str(i)])
            M(j,:)=M(j,:)-((M(j,i)/M(i,i))*M(i,:));
            disp (M)
        end
    end
end
for i=1:n
    disp(['(1/' num2str(M(i,i)) ')B' num2str(i)])
    M(i,:)=(1/M(i,i))*M(i,:);
    disp (M)
end

disp ('Matrik Hasil Eliminasi Gauss')
disp (M)
disp (' Solusi dari SPL adalah')
for i=1:n
    disp(['x' num2str(i) ' = ' num2str(M(i,n+1))])
end
end

```

Linear simultaneous equations which can be solved by this program must form square matrix and have determinant. For example these following linear equation system

Table 2
The System Linear Equation Problems

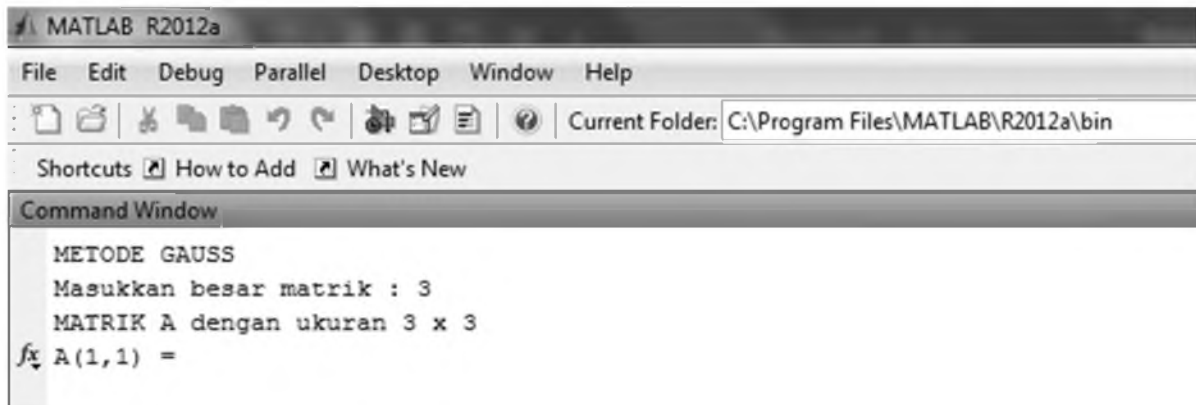
Question 1	Question 2
Solving the Linear Equation System below $3y - 3z = 0$ $x + 4y + 2z = 5$ $3x + 6y + z = 7$	Solving the Linear Equation System below $x + 2y - 3z = 0$ $2x + 4y - 2z = 2$ $3x + 6y - 4z = 3$

This program will be showed to the students of Computer Engineering who take Linear Algebra subject of third semester in 2017-2018 which also will learn the Gauss Jordan elimination. This study conducted an interview with 3 of them which represented from the high, middle and low score of their test. The score used is from Matrix test score in the seventh meeting.

The qualitative research technique which used in this study except making computer program is in-depth interviews. In-depth interviews can be defined as conducting intensive individual interviews with a small number of respondents to explore their perspectives on a particular idea, program or situation (Boyce and Neale, 2006). This study used semi-structured interviews whereas the interviewer prepares a set of same questions to be answered by all interviewees, however, additional questions might be asked during interviews to clarify and/or further expand certain issues. The interview in this study was conducted after the students of Linear Algebra learn about Gauss Jordan Elimination traditionally and matlab program.

4. Result

Linear simultaneous equations given in Table 1 (The System Linear Equation Problems) have been solved with the help of Matlab program and the result is illustrated below



```

MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Program Files\MATLAB\R2012a\bin
Shortcuts How to Add What's New
Command Window
METODE GAUSS
Masukkan besar matrik : 3
MATRIK A dengan ukuran 3 x 3
A(1,1) =

```

Figure 1

The figure above shows how we can input the form of square matrix in the matlab. It can just be used for $n \times n$ matrix. After we input the element of matrix, the program will do elementary row operations to get the reduced-row echelon. It is also showed each elementary row operations that used in the calculation. So, students can learn what kind of elementary row operation used and why it is so.

Table 3
The answer of System Linear Equation Problems using Matlab Program

There is augmented matrix from Linear simultaneous equations that have determinan

```

MATLAB R2012a
File Edit Debug Parallel Desktop Window
Shortcuts How to Add What's New
Command Window
METODE GAUSS
Masukkan besar matrik : 3
Matrik A dengan ukuran 3 x 3
A(1,1) = 0
A(1,2) = 3
A(1,3) = -3
A(2,1) = 1
A(2,2) = 4
A(2,3) = 2
A(3,1) = 3
A(3,2) = 6
A(3,3) = 1
Matrik B dengan ukuran 3 x 1
B(1,1) = 0
B(2,1) = 5
B(3,1) = 7
nilai determinan A = 33.00
PROSES METODE GAUSS
    0    3   -3    0
    1    4    2    5
    3    6    1    7
M =

    1    4    2    5
    0    3   -3    0
    3    6    1    7

B1<--> B2
    1    4    2    5
    0    3   -3    0
    3    6    1    7

```

Whereas, for augmented matrix from Linear simultaneous equations that its determinan is 0,

```

MATLAB R2012a
File Edit Debug Parallel Desktop Wind
Shortcuts How to Add What's New
Command Window
METODE GAUSS
Masukkan besar matrik : 3
Matrik A dengan ukuran 3 x 3
A(1,1) = 1
A(1,2) = 2
A(1,3) = -3
A(2,1) = 2
A(2,2) = 4
A(2,3) = -2
A(3,1) = 3
A(3,2) = 6
A(3,3) = -4
Matrik B dengan ukuran 3 x 1
B(1,1) = 0
B(2,1) = 2
B(3,1) = 3
nilai determinan A = 0.00
Undefined function 'M' for input
arguments of type 'double'.

```



```

B3 - (3/1)B1
  1   4   2   5
  0   3  -3   0
  0  -6  -5  -8

B3 - (-6/3)B2
  1   4   2   5
  0   3  -3   0
  0   0 -11  -8

B2 - (-3/-11)B3
  1.0000   4.0000   2.0000   5.0000
           0   3.0000           0   2.1818
           0           0 -11.0000 -8.0000

B1 - (2/-11)B3
  1.0000   4.0000           0   3.5455
           0   3.0000           0   2.1818
           0           0 -11.0000 -8.0000

B1 - (4/3)B2
  1.0000           0           0   0.6364
           0   3.0000           0   2.1818
           0           0 -11.0000 -8.0000

(1/1)B1
  1.0000           0           0   0.6364
           0   3.0000           0   2.1818
           0           0 -11.0000 -8.0000

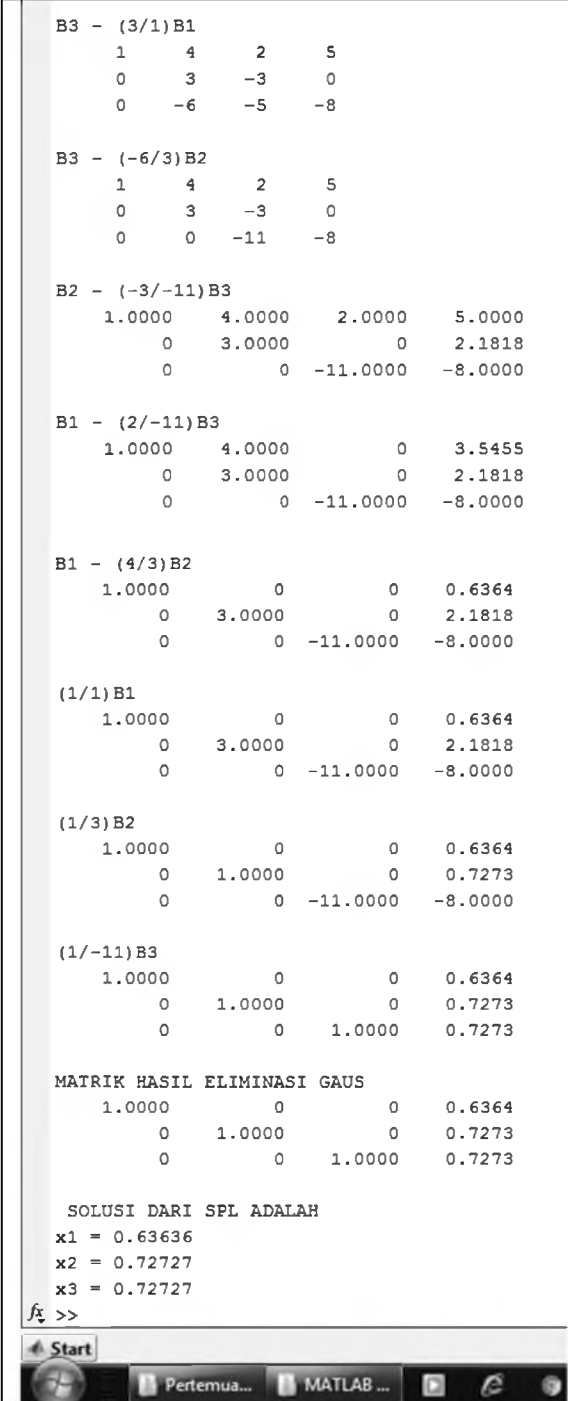
(1/3)B2
  1.0000           0           0   0.6364
           0   1.0000           0   0.7273
           0           0 -11.0000 -8.0000

(1/-11)B3
  1.0000           0           0   0.6364
           0   1.0000           0   0.7273
           0           0   1.0000   0.7273

Matrik Hasil Eliminasi Gauss
  1.0000           0           0   0.6364
           0   1.0000           0   0.7273
           0           0   1.0000   0.7273

SOLUSI DARI SPL ADALAH
x1 = 0.63636
x2 = 0.72727
x3 = 0.72727
fx >>

```



To be used in teaching – learning process, students can be asked to analyze what the change and why it must be changed. Three students were interviewed individually to see what they have understood from the Gauss Jordan elimination using matlab.

Here is the transcript of the high score student about the question number 1 (see table 1.1)

Teacher : Could you see the difference between first and second Matrix?

Student : Yes,

Teacher : What?

Student : The first row is become second row and the second row become the first row.

Teacher : Why it is so?

Student : To make the element of the first row and first column become 1.
Teacher : How about the second row? Is there something you can conclude about it?
Student : Yeah the first element is 0.
Teacher : So?
Student : So what?
Teacher : Did you notice that when the first element of the second row is 0, so what] will you do next?
Student : Uhhmmm yes...that's why the next operation is $B_3 - (3/1)B_1$.
Teacher : What do you get?
Student : $B_3 - (3/1) B_1$ means that the third row must be subtracted by three times of first row. This make the column become pivotal 1.
Teacher : Great Job ,..... so can you conclude the reason of the next operation of the Gauss Jordan elimination in this program?
Student : Yeah....I will try.
Teacher : Good....

From the interview with the high score, she can easily catch the reason of the elementary row operation to get the gauss-jordan methods. She thought that it was really helpful to understand the way to do Gauss Jordan method and also to exercise. She think to use this matlab program to check her answer when doing elementary row operation due to she often get stuck when finding what kind of row operation needs to make the matrix become reduced echelon-row.

Meanwhile, the second interviewee, the middle score, gave different finding. He could understand the way and the reason of each step in the Gauss-Jordan methods. But he still does not confident to use his own way to get the elementary row-operation. He tent to use the same way (the same row-operation) when solving new problem, but when it was does not work he need long time to get the right one.

Furthermore, the last interviewee, the low score, said that it was wonderful to have this kind of program. For him, this program has changed their opinion about Gauss-Jordan method. He thought that if the computer could do such thing, he as a human, should be cleverer. But he still had to be guided to know the reason of some row-operation. Also, he still does not confidence to make his own row operation when doing other question. Therefore, he always asked whether his row operation was correct or not. But when he was asked to refer to the previous method that showed in the matlab program, slow but sure he could choose his own row-operation to solve the problem.

To conclude, the matlab program that used in this study was felt helpful by the student as a guidance to solve system of linear equation with Gauss-Jordan method. They said that they can use it as a replacement of teacher at home. But the problem is students have to install Matlab in their computer/laptop.

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